

Can Born Infeld Gravity Explain Galaxy Rotation Curves?

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Abstract

A Born-Infeld theory has been proposed in order to explain both dark matter and dark energy. We show that the approximation used to deduce the flat rotation curves in this model breaks down at large distances in a spherically symmetric configuration.

1 Introduction

Dark matter and dark energy are among the most exciting topics for cosmology and astrophysics. The first was introduced in order to account for the fact that the velocities of particles and bodies in galaxies do not fall with distance as they should if the only matter source present was visible. Dark energy on the other side can provide the necessary ingredient to the understanding of the late time acceleration of the universe.

It has been pointed out that actually, in both cases the discrepancy between observations and theory could be the signal that something is missing in our knowledge of gravity [1, 2, 3, 4, 5, 6]. This path is philosophically different from the current trend which favors new particle physics beyond the standard model [7, 8, 9, 10, 11]. It has led to different proposals, each with its own successes and shortcomings [12, 13, 14, 15].

The interest of the proposal [16] is that, if correct it solves both problems simultaneously. One can question the minimality and motivation of the model. In the popular trend (Lambda CDM cosmology), one uses fields very similar to ordinary matter but displaying very small interactions with it. It is especially appealing to begin with supersymmetry and obtain as a by product the explanation for the rotation curves of galaxies. The model proposed in [16] introduces

a new symmetric second rank tensor. It does not couple directly to ordinary matter but interacts with the metric, just like dark matter. A more profound motivation was sketched.

In this note we analyze the first order approximation used in [16] which led to the claim that this theory could explain the flat rotation curves observed in galaxies. We essentially point out that the approximation generically breaks down far from the source, because the first non vanishing contribution of the expansion is bigger than the zeroth one in that region. One thus needs a more careful analysis of the problem, which is not done in this short letter.

2 The model

One considers the universe to be governed by the following Lagrangian

$$S = \frac{1}{16\pi G} \int d^4x \left[\sqrt{|g|} R + \frac{2}{\alpha l^2} \sqrt{|g_{\mu\nu} - l^2 K_{\mu\nu}|} + L^{matter}(\psi, g) \right] \quad . \quad (1)$$

The Riemannian metric is denoted as usual by $g_{\mu\nu}$; G is the Newton constant. There is a new length scale l and a dimensionless constant α in the theory. The tensor $K_{\mu\nu}$ is the curvature of a new connection $C_{\nu\rho}^\mu$. This connection is the Christoffel symbol of a new symmetric second rank tensor which is denoted $q_{\alpha\beta}$ while its inverse is written $q^{\alpha\beta}$.

Notice that ordinary, baryonic matter couples directly only to gravity. Its link to the new field is an indirect one. The equations of motion in the gravity sector are

$$G_{\mu\nu} = -\frac{1}{l^2} \sqrt{\frac{q}{g}} g_{\mu\alpha} q^{\alpha\beta} g_{\beta\nu} + 8\pi G T_{\mu\nu}^m \quad \text{and} \quad K_{\mu\nu} = \frac{1}{l^2} (g_{\mu\nu} + \alpha q_{\mu\nu}) \quad . \quad (2)$$

In the absence of ordinary matter, one can find solutions for which there is a constant factor between the two symmetric tensor fields. The preceding equations then formally look like usual Einstein gravity with a cosmological constant. De Sitter space time is therefore a solution and this model can explain the late time acceleration of a Robertson-Walker space time.

3 The galactic rotation curves

For a spherically symmetric configuration, the approach taken in [16] is much more involved. The metric and its companion tensor are taken to be of the form

$$\begin{aligned}
ds^2 &= -c^2 \left(1 + \frac{1}{c^2} \Phi(r)\right) dt^2 + \left(1 - \frac{2m(r)}{c^2 r}\right)^{-1} dr^2 + r^2 d\Omega^2, \\
q_{\mu\nu} dx^\mu dx^\nu &= -\beta^2 c^2 \left(1 - \frac{\omega_0}{\tilde{k}(r)}\right) dt^2 + \left(1 - \frac{\omega_0}{\tilde{k}(r)}\right)^{-1} (\tilde{k}'(r))^2 dr^2 \\
&\quad + (\tilde{k}(r))^2 d\Omega^2.
\end{aligned} \tag{3}$$

The expansion is made in terms of the new length scale:

$$\begin{aligned}
\Phi(r) &= \Phi^{(0)}(r) + \frac{1}{l^2} \Phi^{(1)}(r) + \frac{1}{l^4} \Phi^{(2)}(r) + \dots, \\
m(r) &= m^{(0)}(r) + \frac{1}{l^2} m^{(1)}(r) + \frac{1}{l^4} m^{(2)}(r) + \dots.
\end{aligned} \tag{4}$$

The approximation found in [16] was obtained by the following procedure. One first excludes all normal matter since the new field is supposed to play the role of dark energy, which dominates over ordinary matter. Thus the baryonic energy momentum tensor in the first part of Eq(2) is taken to be vanishing. Secondly, one considers that the length l being of cosmological scale, the right hand side of the second part of Eq(2) can be neglected in the first approximation when dealing with a galaxy. The metric can then be taken to be the Schwarzschild solution with an arbitrary mass, but the vanishing mass was chosen instead. This means one has

$$\Phi^{(0)}(r) = m^{(0)}(r) = 0. \tag{5}$$

The form given to the second tensor is a solution to the field equations given in Eq(2), in the limit $l \rightarrow \infty$. It works for arbitrary values of the parameters ω_0, β and for an arbitrary function $\tilde{k}(r)$, which is not specified at this point. We will come back to this later. One then writes the equations linking the unknown functions $\Phi^{(1)}(r), m^{(1)}(r)$ and $\tilde{k}(r)$ which till now is arbitrary. Making a change of variables, one obtains the following expressions:

$$\begin{aligned}
r(k) &= A_0 \left(- \left(k - \frac{1}{2} \right) \ln \left(1 - \frac{1}{k} \right) - 1 \right) + B_0 \left(k - \frac{1}{2} \right), \\
u^{(1)}(k) &= \frac{1}{2} \beta c^2 B_0 \left[A_0 \left(k^2 \left(1 - \frac{1}{k} \right) \ln \left(1 - \frac{1}{k} \right) + k - \frac{1}{2} \right) - B_0 (k^2 - k) \right], \\
m^{(1)}(k) &= \frac{\omega_0^3 c^2}{2\beta} \left(\frac{1}{3} k^3 + \frac{1}{2} k^2 + \ln(k-1) - h_0 \right),
\end{aligned} \tag{6}$$

where the function $u^{(1)}(r)$ has been defined by the relation

$$r \frac{d\Phi^{(1)}(r)}{dr} = u^{(1)}(r) + \frac{m^{(1)}(r)}{r}. \tag{7}$$

According to the values of the parameters A_0, B_0, ω_0 , the solution can display different behaviors. We shall in this paper confine ourselves to the so called

"logarithmic branch" for which one has $A_0 > 0, B_0 < 0$. Under these conditions, the function $r(k)$ evolves between two extreme values: a finite value k_0 where it vanishes and the value $k = 1$ where it diverges.

Looking at Eq(6) , one sees that the radial coordinate r goes to infinity as k goes to one. We now want to derive an approximation of the different components of the metric in terms of the radius in the asymptotic region. Near $k = 1$, one can replace k by unity everywhere, except in the term containing the logarithm since it blows up. Inverting the relation, one readily finds

$$\tilde{k}(r) = \omega_0 \left[1 - \exp \left(-2 + \frac{B_0}{A_0} \right) \exp \left(-\frac{2}{A_0} r \right) \right]^{-1} . \quad (8)$$

Using the same argument, one obtains for the other functions the following expressions:

$$\begin{aligned} u^{(1)}(r) &= \frac{1}{2} \beta c^2 \omega_0 A_0 \left\{ \exp \left(-2 + \frac{B_0}{A_0} \right) \exp \left(-\frac{2}{A_0} r \right) \frac{2}{A_0} \left(-r - A_0 + \frac{1}{2} B_0 \right) + \frac{1}{2} \right\} , \\ m^{(1)}(r) &= \frac{\omega_0^3 c^2}{2\beta} \left[\frac{11}{6} - h_0 + \frac{2}{A_0} \left(-r - A_0 + \frac{1}{2} B_0 \right) \right] . \end{aligned} \quad (9)$$

The form of the rotation curves of galaxies is fixed by the Newtonian potential via the equation

$$v(r) = \sqrt{r \frac{d\Phi(r)}{dr}} , \quad (10)$$

where $v(r)$ is the velocity of an object lying at a distance r from the center. Using the expressions obtained in Eq(9) and the Formula displayed in Eq(7) , one obtains in the asymptotic region ($r \rightarrow \infty$) the approximation

$$\begin{aligned} \frac{d\Phi^{(1)}(r)}{dr} &= \frac{K_1}{r} + \frac{K_2}{r^2} \quad \text{with} \\ K_1 &= -\frac{\omega_0^3 c^2}{2\beta} \frac{2}{A_0} + \frac{1}{4} \beta c^2 \omega_0 A_0 \quad \text{and} \quad K_2 = \frac{\omega_0^3 c^2}{2\beta} \left(\frac{11}{6} - h_0 + \frac{2}{A_0} \right) \end{aligned} \quad (11)$$

Some exponential terms, being subdominant in the asymptotic region, have been neglected. The flatness of the rotation curves is due to the fact that the first term in the last equation is the dominant one at large distances; K_1 is the velocity at infinity. This result was obtained in [16] . Our derivation simply changes the point of view, writing everything in terms of the physical radius. One may remark that the constant K_2 is nothing but the Schwarzschild mass ; dropping it in the zeroth order approximation does not prevent it from coming back in the next one. This is a generic feature when one solves perturbatively a differential equation. In the very far region, this term will also be dropped.

We simply want to point out that the approximation obtained so far is actually not reliable. When the first non vanishing contribution to a perturbative expansion is bigger than the background, one has to go to higher orders to draw

a solid conclusion. To be precise, let us look at the time-time component of the metric. Stopping at the first order one has

$$g_{00} = -c^2 \left(1 + \frac{1}{c^2 l^2} \Phi^{(1)}(r) \right) \quad , \quad \Phi^{(1)}(r) = K_1 \ln \frac{r}{r_0} \quad , \quad (12)$$

r_0 being an arbitrary constant. Working with this approximation is legitimate only when

$$|\Phi^{(1)}(r)| \ll c^2 l^2 \quad \text{i.e.} \quad r \ll r_\star = r_0 \exp \left(\frac{c^2 l^2}{K_1} \right) \quad . \quad (13)$$

This simple calculation shows that the approximation obtained so far is valid only at a finite distance from the source. Although one may choose the parameters to fit some rotation curves, the problem is that one is never truly in the asymptotic region. There is no guarantee that beyond r_\star the velocities will have the same behavior. We make more comments and remarks in the conclusions.

In the same way

$$g_{rr}^{-1} = \left(1 - \frac{2}{c^2 r} \frac{1}{l^2} m^{(1)}(r) \right) \quad . \quad (14)$$

is accurate only when the second term is small compared to the first. This leads to the relation

$$\frac{2\omega_0^3}{\beta l^2 A_0} \ll 1 \quad , \quad (15)$$

where we have neglected terms depending on the radial coordinate which vanish at infinity. This second constraint is odd since it would imply that the free parameters of the solutions (A_0, ω_0) of the model can not be chosen at will.

4 Conclusions

We have seen that the perturbative approximation used to claim that Born Infeld gravity could explain flat galaxy rotation curves without dark matter is problematic. Firstly, it breaks down at infinity. This could be anticipated because one needs a Newtonian potential which scales like a logarithm of the distance in the asymptotic region. Obtaining this kind of behavior at first order with a zeroth order in which that potential vanishes everywhere (the mass was taken to be zero) is clearly not possible. The second point is that it imposes unnatural conditions on the free parameters of the solution describing a spherically symmetric configuration.

Can a higher order approximation solve the problem? This remains to be seen. One would have to be careful and in principle expand also the function $\tilde{k}(r)$ in terms of the new length scale. This was not done in [16]; the fact that there were three equations of motion for the functions $\Phi^{(1)}(r), m^{(1)}(r), \tilde{k}(r)$ may be interpreted as assuming $\tilde{k}^{(0)}(r)$ to also be vanishing so that the function computed was actually $\tilde{k}^{(1)}(r)$. This choice is of course singular and would have

to be changed. The approximation of order n would then be "reliable" to draw conclusions about rotation curves if in the asymptotic region the contribution of order $n + 1$ was subdominant when compared to the sum of its predecessors.

In fact, the situation is even more complicated because the configuration of Eq(3) for the second tensor $q_{\mu\nu}$ is nothing but the Schwarzschild solution. This will remain true no matter which order one considers for the function $\tilde{k}(r)$. It would be surprising that the perturbations affect only the metric field and not its companion. This is a real question which will have to be addressed.

To finish, let us come back to the fact that de Sitter space time is an exact solution of the theory in the absence of matter. If it is the only one, then this model can not explain the rotation curves. But one knows that in the modified theory of gravity studied by [12] for example, the de Sitter and the anti de Sitter spaces are both solutions in vacuum. Something similar may still salvage the claim here.

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